Problem Sheet 2

CMPE 58R, Statistical Data Analysis

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Due: 15 Jul 2011, 10:00.

- **What to submit**
  - A front page with your name, date of submissions and the list of the questions that you have solved. An example front page is given as the last page of this assignment sheet. Obeying the format would save my time.
  - Don't send any executables, or source code. The output of an example run is sufficient.
- **How and when to submit**
  - Hand in before the class on the day of the deadline.
  - We accept printed or handwritten solutions. Use preferably both sides of pages.
- **Grading**
  - Each question is graded over 1. You get separate grades for each question.

1. (**Coin**) Suppose a biased coin with $p(\text{head}) = \pi$ is thrown N times. The number of times head shows up is denoted by x. We observe that $x = 4$. Assume π and N are unknown. Assuming flat priors $p(N)$ and $p(\pi)$, answer the following via analytic derivation or computation:

   (a) How many times is the coin thrown in total (marginal MAP estimate of N derived from $p(N|x)$) ?

   (b) What is the probability that head shows up (MAP estimate of π) ?

   (c) Write a R or Matlab program to plot $p(N, \pi|x)$ (in Matlab you can use `imagesc`) and the marginals $p(N|x)$ and $p(\pi|x)$.

2. (**Gaussian**) Suppose we observe the following data set $X = \{1, 2, 3, 5\}$. Assume $x_i \propto \mathcal{N}(x_i; \mu, s)$, $\mu \sim \mathcal{N}(\mu; 0, 100)$ and $s \sim \mathcal{IG}(s; 1, 1)$. Define $\hat{s}$ as the sample variance of X. Define the posterior densities

$$\begin{aligned} \pi_k(\mu) &= p(\mu|x_{1:k}) \\ f_k(\mu) &= p(\mu|x_{1:k}, s = \hat{s}) \end{aligned}$$

   where $x_{1:k} = \{x_1, x-2, \dots, x_k\}$. Derive and plot these densities for $k = 1 \dots 4$.

3. (**Forged?**) Suppose we have a coin that we throw $N = 20$ times. Each outcome is independent Bernoulli with probability of tails π. We observe 4 tails in 20 throws. What is the probability that this is a forged coin?

   [Hint: Assume a model indicator M with two values M_0 and M_1. If $M = M_0$ then the coin is not forged and assume each outcome is generated according to a Bernoulli $p(x_i|\pi = 0.5, M = M_0)$. If the coin is forged, assume the probability of tails π is uniformly random with a beta prior $\mathcal{B}(1, 1)$ and compute $p(x_i|M = M_1)$. Compute the probability that the coin is forged, conditioned on the observations $x_1, \dots, x_{20}$.]

4. (**Number of dice**) We throw N dice and observe that their sum is 12. How many dice were thrown?