



Public Key Cryptography

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What is Cryptography

- ▶ Cryptography is the study of secure communications techniques that allow only the sender and intended recipient of a message to view its contents.

(Kaspersky)

Type of Cryptography

- ▶ Symmetric Key Cryptography
 - ▶ Fast
 - ▶ Key Exchange is risky
- ▶ Asymmetric Key Cryptography (Public Key Cryptography)
 - ▶ Slow
 - ▶ Key Exchange is not risky

Public Key Cryptography Use Cases

- ▶ Digital Signature
- ▶ Key Exchange
- ▶ Blockchain
- ▶ Public Key Infrastructure (PKI)

Symmetric Key Cryptography

Sender

Key
Message

Receiver

Key

Symmetric Key Cryptography



Public Key Cryptography

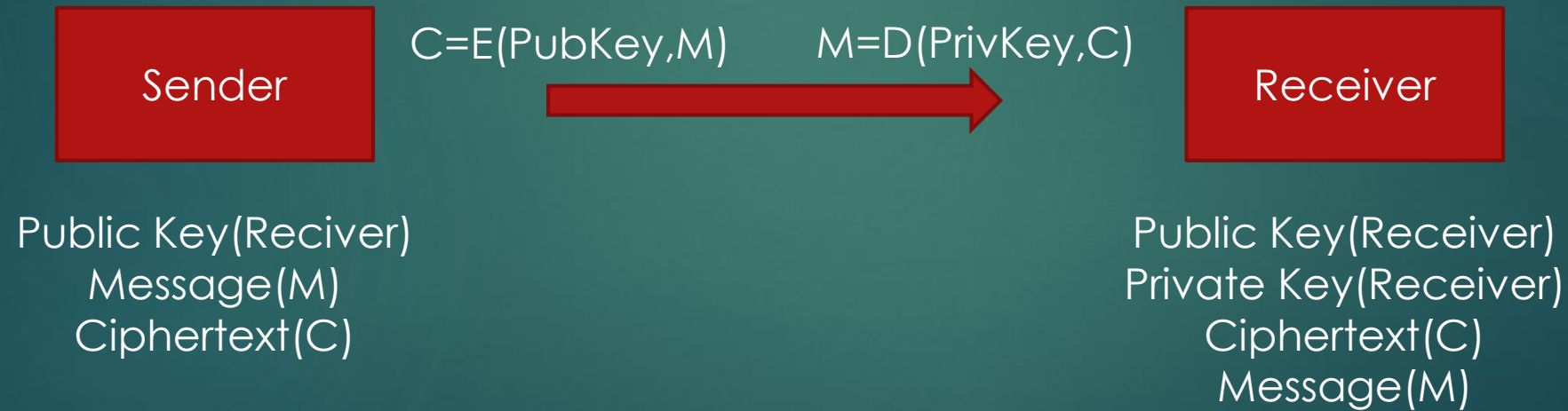
Sender

Public Key(Receiver)

Receiver

Public Key(Receiver)
Private Key(Receiver)

Public Key Cryptography





RSA (Rivest– Shamir–Adleman)

Phases of RSA

- ▶ Generating Keys
- ▶ Encryption and Decryption

Generating Keys

1. Choose two distinct prime numbers 'p' and 'q'
2. Compute 'n = p * q'
3. Compute Euler's Totient $\varphi(n)$

$$\varphi(n) = |\{k: 1 \leq k \leq n, \gcd(n,k) = 1\}|$$

For numbers that multiplication of two relatively prime number like n ;

$$\begin{aligned}\varphi(n) &= p*q - p - q + 1 \\ &= (p-1)*(q-1)\end{aligned}$$

Example: $n=3*5=15$ $\varphi(15) = 15 - 5 - 3 + 1 = 8$

3, 6, 9,12,15 and 5,10,15

$$k = \{1,2,4,7,8,11,13,14\}$$

Generating Keys

4. Choose a public exponent e (Fermat Primes) $(10001)_2 = 65537$
 $e < \varphi(n)$ and $\gcd(e, \varphi(n))=1$
5. Compute a private exponent d (Extended Euclidean Algorithm)
 $ed \equiv 1 \pmod{\varphi(n)}$

Public Key

n
 e (public exponent)

Private Key

d (private exponent)
 $p, q, \varphi(n), n$

Encryption and Decryption

1. Sender takes public key of receiver (n, e)
2. Sender encrypts message

$$0 < m < n$$

$$c = E(m) = m^e \pmod{n}$$

3. Receiver receives ciphertext
4. Receiver decrypts ciphertext

$$m = D(c) = c^d \pmod{n} = m^{ed} \pmod{n}$$

Proof of Decryption Step

- ▶ This proof works only when $\gcd(m, n) = 1$
- ▶ $D(c) = m^{ed} \pmod{n}$
 - ▶ $ed \equiv 1 \pmod{\varphi(n)}$
 - ▶ $ed = 1 + k * \varphi(n)$
- ▶ $D(c) = m^{(1 + k * \varphi(n))} \pmod{n}$
- ▶ $D(c) = m * m^{(k * \varphi(n))} \pmod{n}$
 - $m^{\varphi(n)} \equiv 1 \pmod{n}$ (Euler-Fermat Theorem)
- ▶ $D(c) = m * 1^k \pmod{n}$
- ▶ $D(c) = m$

References

- ▶ <https://www.kaspersky.com/resource-center/definitions/what-is-cryptography>
- ▶ <https://blog.keyfactor.com/symmetric-vs-asymmetric-encryption>
- ▶ https://www.di-mgt.com.au/rsa_theory.html
- ▶ https://www.youtube.com/watch?v=reH9zrGcXXM&ab_channel=ProfessorMacauley