

CMPE 350 - Summer 2014  
PS#5

08.08.14

### Chapter 3

3.10 A Turing machine with doubly infinite tape is similar to an ordinary Turing machine, but its tape is infinite to the left as well as to the right. The tape is initially filled with blanks except for the portion that contains the input. Computation is defined as usual except that the head never encounters an end to the tape as it moves leftward. Show that this type of Turing machine recognizes the class of Turing-recognizable languages.

3.12 A Turing machine with left reset is similar to an ordinary Turing machine, but the transition function has the form

$$\delta : Q \times \Gamma \rightarrow Q \times \Gamma \times \{R, \text{RESET}\}.$$

If  $\delta(q, a) = (r, b, \text{RESET})$ , when the machine is in state  $q$  reading an  $a$ , the machine's head jumps to the left-hand end of the tape after it writes  $b$  on the tape and enters state  $r$ . Note that these machines do not have the usual ability to move the head one symbol left. Show that Turing machines with left reset recognize the class of Turing-recognizable languages.

3.13 A Turing machine with stay put instead of left is similar to an ordinary Turing machine, but the transition function has the form

$$\delta : Q \times \Gamma \rightarrow Q \times \Gamma \times \{R, S\}.$$

At each point the machine can move its head right or let it stay in the same position. Show that this Turing machine variant is not equivalent to the usual version. What class of languages do these machines recognize?

3.18 Show that a language is decidable iff some enumerator enumerates the language in lexicographic order.

3.19 Show that every infinite Turing-recognizable language has an infinite decidable subset.

### Chapter 4

4.12 Let  $A = \{ \langle R, S \rangle \mid R \text{ and } S \text{ are regular expressions and } L(R) \subseteq L(S) \}$ . Show that  $A$  is decidable.

4.19 Let  $S = \{ \langle M \rangle \mid M \text{ is a DFA that accepts } w^R \text{ whenever it accepts } w \}$ . Show that  $S$  is decidable.

4.22 A useless state in a pushdown automaton is never entered on any input string. Consider the problem of determining whether a pushdown automaton has any useless states. Formulate this problem as a language and show that it is decidable.

- Show that the set of undecidable languages are closed under complementation.