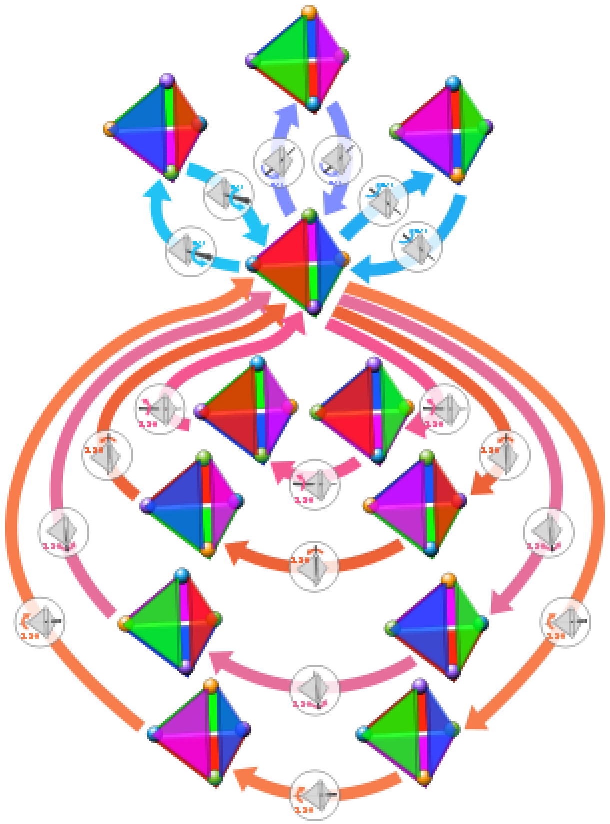
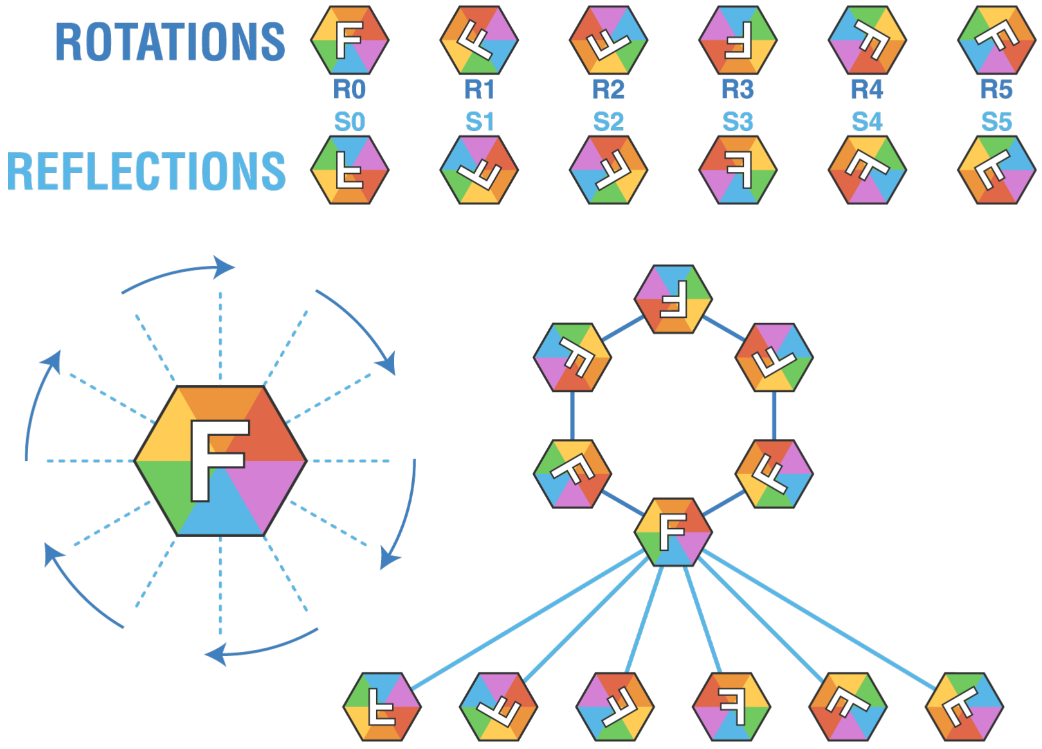
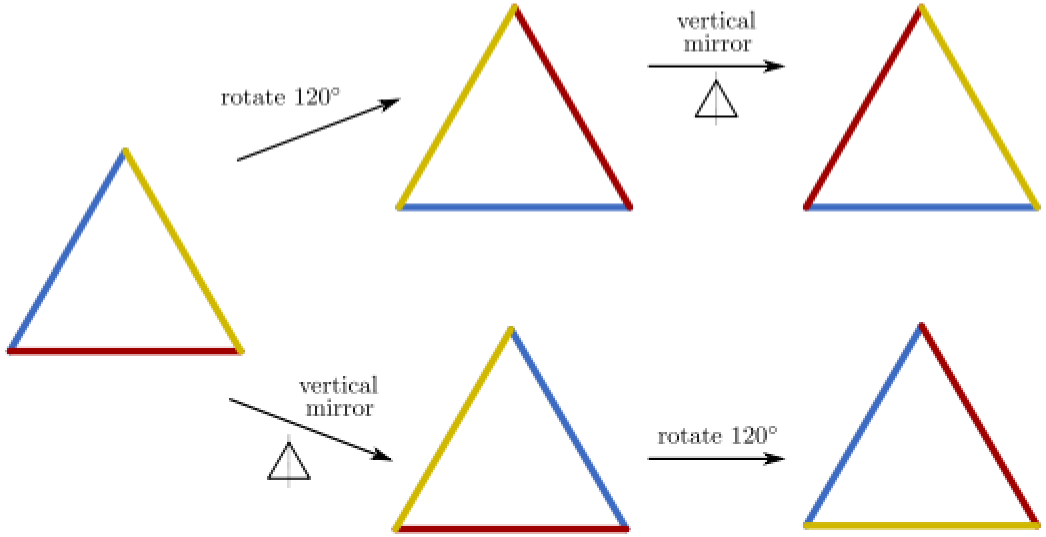




SYMMETRY GROUPS

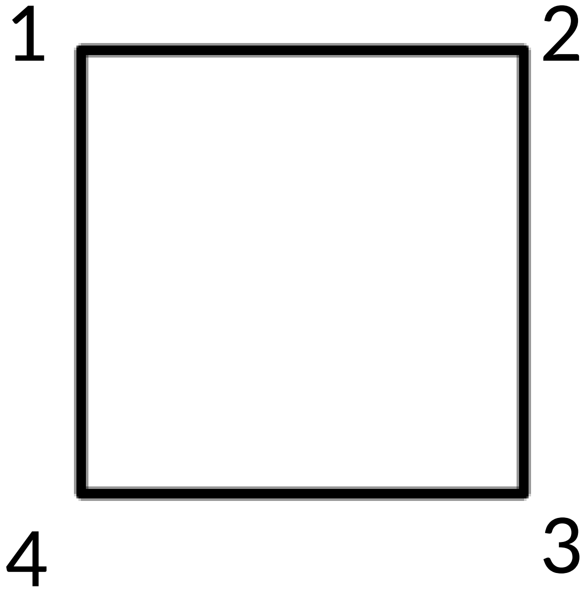
Orhun Görkem

In group theory, the symmetry group of a geometric object is the group of all transformations under which the object is invariant, endowed with the group operation of composition.



Symmetry groups can be in various dimensions.

Rotations and symmetries of a square form a subgroup in 2D space.


1
2
3
4

- Identity operation(Rotation of 0 degrees) R0 $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 \end{bmatrix}$
- Rotation of 90 degrees R1 $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 1 \end{bmatrix}$
- Rotation of 180 degrees R2 $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 3 & 4 & 1 & 2 \end{bmatrix}$
- Rotation of 270 degrees R3 $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 4 & 1 & 2 & 3 \end{bmatrix}$

- Flip about the axis from vertices 1 to 3 R4 $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 1 & 4 & 3 & 2 \end{bmatrix}$
- Flip about the axis from vertices 2 to 4 R5 $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 3 & 2 & 1 & 4 \end{bmatrix}$
- Flip about the vertical axis R6 $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \end{bmatrix}$
- Flip about the horizontal axis R7 $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 4 & 3 & 2 & 1 \end{bmatrix}$

ELEMENTS OF GROUP ARE SYMMETRY AND ROTATION OPERATIONS.
THE BINARY OPERATION IS COMPOSITION OF SYMMETRIES AND ROTATIONS.

R1 o R1 = $\begin{bmatrix}1 & 2 & 3 & 4\\ 2 & 3 & 4 & 1\end{bmatrix}$ o $\begin{bmatrix}1 & 2 & 3 & 4\\ 2 & 3 & 4 & 1\end{bmatrix}$ = R2 = $\begin{bmatrix}1 & 2 & 3 & 4\\ 3 & 4 & 1 & 2\end{bmatrix}$

Ri o Rj = Rk where i,j,k ∈ {0,1,2,3,4,5,6,7} Closedness (Operation of composition stays in group.)

R1 o (R2 o R6) = (R1 o R2) o R6 = R5 Associativity

R0 = e = identity element

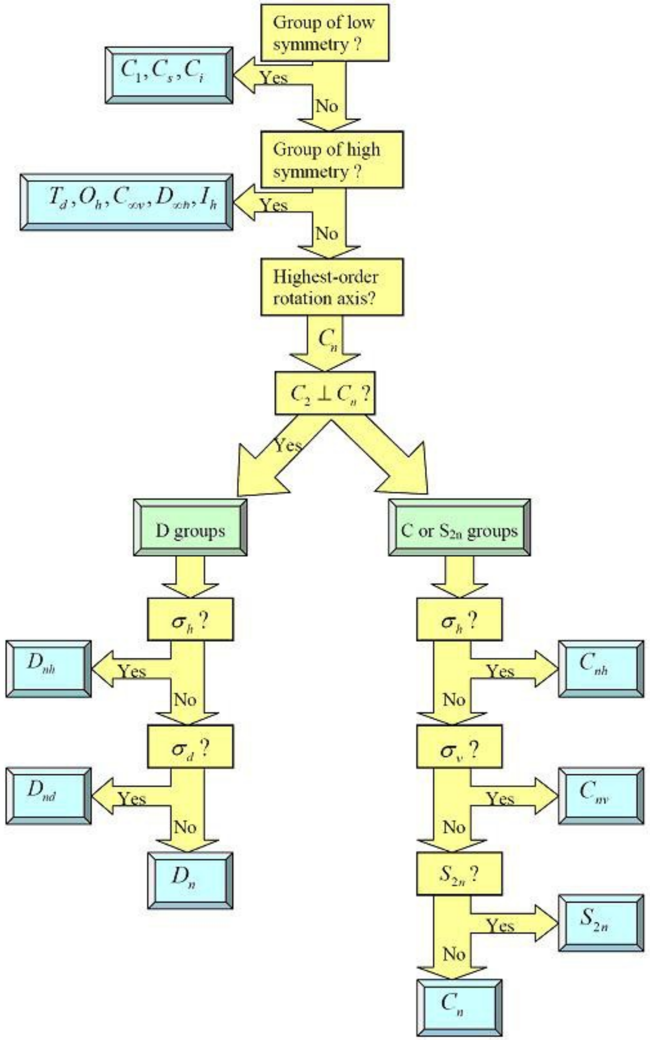
Ri o Ri = R0 where i ∈ { 4,5,6,7 } and Ri o R(4-i) =R0 where i ∈ { 1,2,3 }

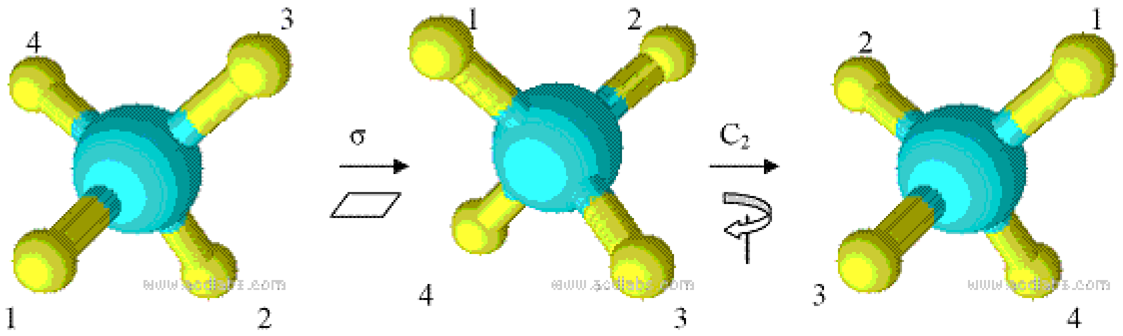
R1 o R7 = R5 != R4 = R7 o R1 Not commutative

A dihedral group is a group that can be “generated” by combining a rotation symmetry and a mirror reflection multiple times. A dihedral group with n rotational and n mirror symmetries is commonly named Dn. We can not reach each element of this group by combining a single operation multiple times.

R0, R1, R2, R3 and the operation of composition form a subgroup but that group is not dihedral.

APPLICATION IN CHEMISTRY





All the symmetry operations of a molecule as a group can be written in the form of group multiplication table and they obey all the properties of a group. This group is called symmetry point group. To determine the symmetry point group of a molecule is very important, because all symmetry related properties are dependent on the symmetry point group of the molecule.

Types of Point Groups

- Nonaxial
- Cyclic
- Dihedral
- Polyhedral
- Linear

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