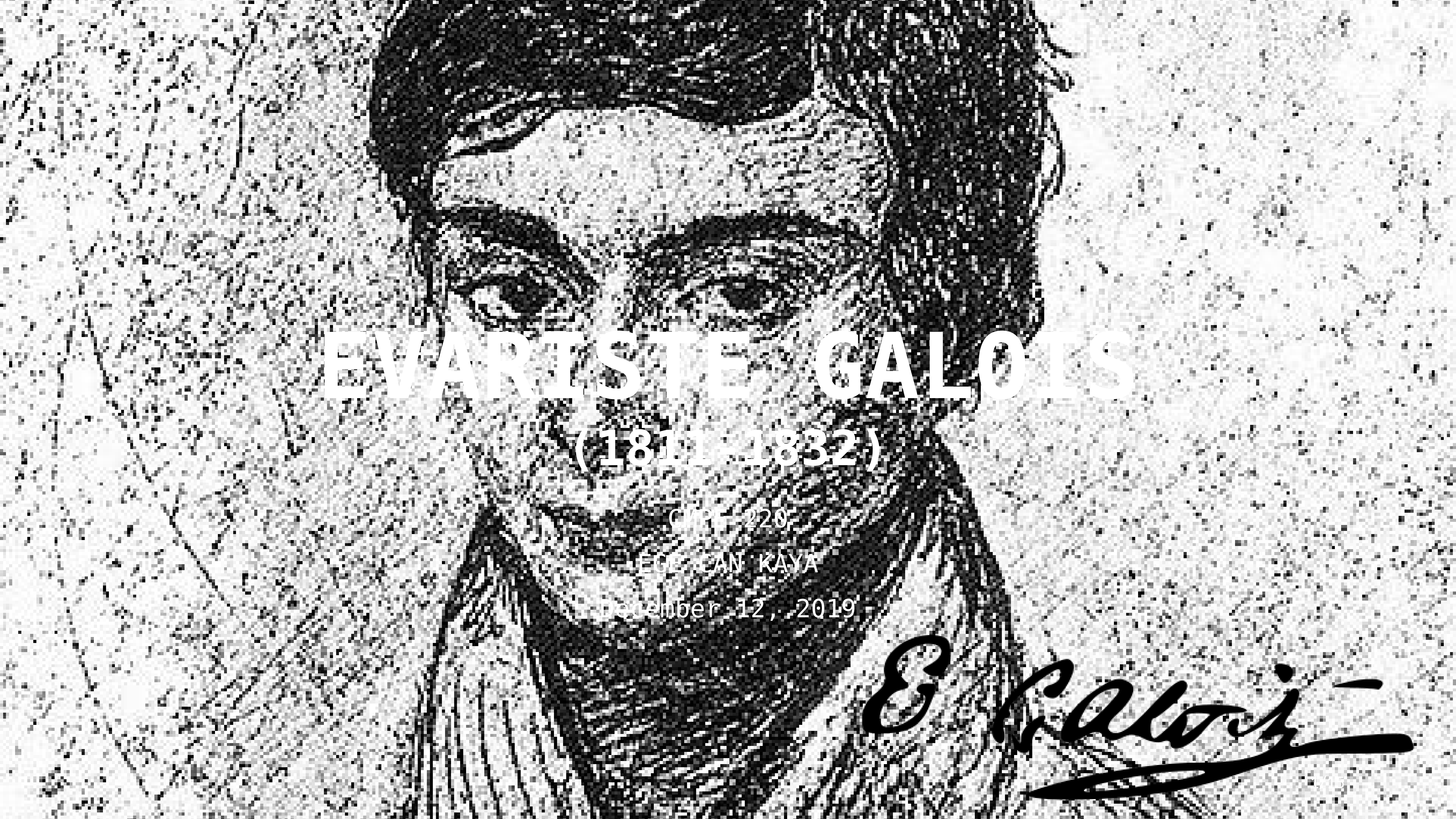

EVARISTE GALOIS

(1811-1832)

CMPE 220

EGE CAN KAYA

December 12, 2019

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EARLY LIFE

- Historical context: France in political turmoil (Republicans vs Royalists)
  - French Revolution (1789)
  - French First Republic (1792-1804)
  - Napoleon becomes emperor: First French Empire (1804-1814)
  - Restoration of the monarchy (1815-1830)
  - And then?
- Galois born in 1811
- His father: a Republican & the mayor of his village

- Becomes “seriously interested” in mathematics at 14
- Reads Legendre and Lagrange’s papers

UNFORTUNATE EVENTS (AGES 17-19)

- Tries to enter Ecole polytechnique, rejected. Goes to far inferior institution, Ecole normale
- A paper that cannot be published:
  - Cauchy (speculation: political views?)
  - Fourier dies
- Political dispute with a priest: Galois' father commits suicide
- Ecole polytechnique: failure again

- 26-29 July 1830: the Three Glorious Days (aka the July Revolution)
- Signed open letter
- Expelled from Ecole normale
- Joins the National Guard, artillery unit
- Unit disbanded, arrested

- A toast to the king: arrested again
- 14 July 1831: head of protest. Arrested again
- Six months in prison
- Political activism becomes the priority
- Poisson: paper finally published?
- Loses faith in the Academy

[illegible] au moyen de ce
[illegible] et réciproquement. Je n'en dirai rien.

Tu sais, mon cher Auguste, que ces sujets ne sont pas les seuls que j'ai explorés. [redacted] Mes principales méditations depuis quelque temps étaient dirigées sur l'application à l'analyse transcendante de la théorie de l'ambiguïté. Il s'agissait de voir a priori dans une relation entre des quantités ou fonctions transcendantes, quels échanges on pouvait faire, quelles quantités on pouvait substituer aux quantités données sans que la relation pût cesser d'avoir lieu. Cela fait reconnaître tout de suite l'impossibilité de beaucoup d'expressions que l'on pourrait chercher. Mais je n'ai pas le temps et mes idées ne sont pas encore bien développées sur ce terrain qui est immense.

Tu feras imprimer cette lettre dans la Revue encyclopédique.

- The closing line of his letter:

“Ask Jacobi or Gauss publicly to give their opinion, not as to the truth, but as to the importance of these theorems. Later there will be, I hope, some people who will find it to their advantage to *decipher all this mess*.”

- “Early in the morning of 30 May 1832, he was shot in the abdomen, abandoned by his opponent (who?) (…) and was found by a passing farmer. He died the following morning at ten o'clock (…) after refusing the offices of a priest. His funeral ended in riots. There were plans to initiate an uprising during his funeral, but this did not occur. Only Galois' younger brother was notified of the events prior to Galois' death. He was 20 years old. His last words to his younger brother Alfred were:

  Don't cry, Alfred! I need all my courage to die at twenty!”

LEGACY & CONTRIBUTIONS TO MATHEMATICS

- His work is finally then reviewed and approved by Liouville
- The 60 pages of his manuscript affect almost every branch of mathematics
- Galois theory: MATH323

quadratic $$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

cubic $$x = \frac{-2b + \left(\frac{-1+\sqrt{-3}}{2}\right)^n \sqrt[3]{4\left(-2b^3 + 9abc - 27a^2d + \sqrt{(-2b^3 + 9abc - 27a^2d)^2 - 4(b^2 - 3ac)^3}\right)} + \left(\frac{-1-\sqrt{-3}}{2}\right)^n \sqrt[3]{4\left(-2b^3 + 9abc - 27a^2d - \sqrt{(-2b^3 + 9abc - 27a^2d)^2 - 4(b^2 - 3ac)^3}\right)}}{6a}$$

quartic $$x = \frac{-3b \pm \left(\sqrt{3\left(3b^2 - 8ac + 2a\sqrt[3]{4\left(2c^3 - 9bcd + 27ad^2 + 27b^2e - 72ace + \sqrt{(2c^3 - 9bcd + 27ad^2 + 27b^2e - 72ace)^2 - 4(c^2 - 3bd + 12ae)^3}\right)} + \right.}\right.}{}$$

$$\overline{2a\sqrt[3]{4\left(2c^3 - 9bcd + 27ad^2 + 27b^2e - 72ace - \sqrt{(2c^3 - 9bcd + 27ad^2 + 27b^2e - 72ace)^2 - 4(c^2 - 3bd + 12ae)^3}\right)}\Big) \pm \sqrt{3\Big(3b^2 - 8ac +}}$$

$$\overline{2a\left(\frac{-1+\sqrt{-3}}{2}\right)\sqrt[3]{4\left(2c^3 - 9bcd + 27ad^2 + 27b^2e - 72ace + \sqrt{(2c^3 - 9bcd + 27ad^2 + 27b^2e - 72ace)^2 - 4(c^2 - 3bd + 12ae)^3}\right)} + 2a\left(\frac{-1-\sqrt{-3}}{2}\right)}$$

$$\frac{\sqrt[3]{4\left(2c^3 - 9bcd + 27ad^2 + 27b^2e - 72ace - \sqrt{(2c^3 - 9bcd + 27ad^2 + 27b^2e - 72ace)^2 - 4(c^2 - 3bd + 12ae)^3}\right)}\Big)\Big) \pm \operatorname{sgn}\Big(\Big(\operatorname{sgn}(-b^3 + 4abc - 8a^2d) - \frac{1}{2}\Big)}{12a}$$

$$\overline{\Big(\operatorname{sgn}(\max((2c^3 - 9bcd + 27ad^2 + 27b^2e - 72ace)^2 - 4(c^2 - 3bd + 12ae)^3, \min(3b^2 - 8ac, 3b^4 + 16a^2c^2 + 16a^2bd - 16ab^2c - 64a^3e))) - \frac{1}{2}\Big)\Big),}$$

$$\overline{\sqrt{3\left(3b^2 - 8ac + 2a\left(\frac{-1-\sqrt{-3}}{2}\right)\sqrt[3]{4\left(2c^3 - 9bcd + 27ad^2 + 27b^2e - 72ace + \sqrt{(2c^3 - 9bcd + 27ad^2 + 27b^2e - 72ace)^2 - 4(c^2 - 3bd + 12ae)^3}\right)} + \right.}}$$

$$\overline{2a\left(\frac{-1+\sqrt{-3}}{2}\right)\sqrt[3]{4\left(2c^3 - 9bcd + 27ad^2 + 27b^2e - 72ace - \sqrt{(2c^3 - 9bcd + 27ad^2 + 27b^2e - 72ace)^2 - 4(c^2 - 3bd + 12ae)^3}\right)}\Big)}$$

- How? An extremely simplified explanation:
  - The formulas for the solutions of all look like towers: using square-roots, square-roots of square-roots, or cube-roots of square-roots, and so on upwards.
  - Galois realized that the numbers that can be reached from these sorts of step-by-step towerings of square-roots or cube-roots or suchlike must all have a quite specific kind of symmetry.
  - We have an example of a quintic polynomial whose solutions have a less rigid symmetry than that kind of specific symmetry which formulas generate.
  - Therefore it must be impossible to represent its solutions in terms of those kinds of formulas.

quadratic

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

cubic

$$x = \frac{-2b + \left(\frac{-1+\sqrt{-3}}{2}\right)^n \sqrt[3]{4\left(-2b^3 + 9abc - 27a^2d + \sqrt{(-2b^3 + 9abc - 27a^2d)^2 - 4(b^2 - 3ac)^3}\right)} + \left(\frac{-1-\sqrt{-3}}{2}\right)^n \sqrt[3]{4\left(-2b^3 + 9abc - 27a^2d - \sqrt{(-2b^3 + 9abc - 27a^2d)^2 - 4(b^2 - 3ac)^3}\right)}}{6a}$$

quartic

$$x = -3b \pm \Bigg(\sqrt{3\Big(3b^2 - 8ac + 2a\sqrt[3]{4\left(2c^3 - 9bcd + 27ad^2 + 27b^2e - 72ace + \sqrt{(2c^3 - 9bcd + 27ad^2 + 27b^2e - 72ace)^2 - 4(c^2 - 3bd + 12ae)^3}\right)} +}$$

$$2a\sqrt[3]{4\left(2c^3 - 9bcd + 27ad^2 + 27b^2e - 72ace - \sqrt{(2c^3 - 9bcd + 27ad^2 + 27b^2e - 72ace)^2 - 4(c^2 - 3bd + 12ae)^3}\right)}\Big) \pm \sqrt{3\Big(3b^2 - 8ac +}$$

$$2a\left(\frac{-1+\sqrt{-3}}{2}\right)\sqrt[3]{4\left(2c^3 - 9bcd + 27ad^2 + 27b^2e - 72ace + \sqrt{(2c^3 - 9bcd + 27ad^2 + 27b^2e - 72ace)^2 - 4(c^2 - 3bd + 12ae)^3}\right)} + 2a\left(\frac{-1-\sqrt{-3}}{2}\right)$$

$$\frac{\sqrt[3]{4\left(2c^3 - 9bcd + 27ad^2 + 27b^2e - 72ace - \sqrt{(2c^3 - 9bcd + 27ad^2 + 27b^2e - 72ace)^2 - 4(c^2 - 3bd + 12ae)^3}\right)}\Big)\Bigg) \pm \operatorname{sgn}\Big(\Big(\operatorname{sgn}(-b^3 + 4abc - 8a^2d) - \frac{1}{2}\Big)}{12a}$$

$$\Big(\operatorname{sgn}(\max((2c^3 - 9bcd + 27ad^2 + 27b^2e - 72ace)^2 - 4(c^2 - 3bd + 12ae)^3, \min(3b^2 - 8ac, 3b^4 + 16a^2c^2 + 16a^2bd - 16ab^2c - 64a^3e))) - \frac{1}{2}\Big)\Big),$$

$$\sqrt{3\Big(3b^2 - 8ac + 2a\left(\frac{-1-\sqrt{-3}}{2}\right)\sqrt[3]{4\left(2c^3 - 9bcd + 27ad^2 + 27b^2e - 72ace + \sqrt{(2c^3 - 9bcd + 27ad^2 + 27b^2e - 72ace)^2 - 4(c^2 - 3bd + 12ae)^3}\right)} +}$$

$$2a\left(\frac{-1+\sqrt{-3}}{2}\right)\sqrt[3]{4\left(2c^3 - 9bcd + 27ad^2 + 27b^2e - 72ace - \sqrt{(2c^3 - 9bcd + 27ad^2 + 27b^2e - 72ace)^2 - 4(c^2 - 3bd + 12ae)^3}\right)}\Big)}$$

- Also, contributions to group theory and analysis (Abelian integrals, continued fractions)

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