CMPE 220.02

DISCRETE COMPUTATIONAL STRUCTURES

General Formula for Boolean Functions of n Variables

by

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p	$f_0 = F$	$f_1 = p$	$f_2 = \neg p$	$f_3 = T$
F	F	F	T	T
T	F	T	F	T

TABLE 1. Boolean functions of one variable

p	q	$f_0 = F$	$f_1 = p \wedge q$	$f_2 = \neg(p \longrightarrow q)$	$f_3 = p$	f_4	$f_5 = q$	$f_6 = p \oplus q$	$f_7 = p \vee q$	$f_8 = p$ NOR q	$f_9 = p \longleftrightarrow q$	$f_{10} = \neg q$	f_{11}	$f_{12} = \neg p$	$f_{13} = p \longrightarrow q$	$f_{14} = p$ NAND q	$f_{15} = T$
F	F	F	F	F	F	F	F	F	F	T	T	T	T	T	T	T	T
F	T	F	F	F	F	T	T	T	T	F	F	F	F	T	T	T	T
T	F	F	F	T	T	F	F	T	T	F	F	T	T	F	F	T	T
T	T	F	T	F	T	F	T	F	T	F	T	F	T	F	T	F	T

TABLE 2. Boolean functions of two variables

from Discrete Mathematics Lecture Handouts of Haluk Bingöl

Number of variables	Number of rows	Number of functions
1	2	4
2	4	16
3	8	256
…	…	…
n	2^n	$2^{(2^n)}$

2.3 Proof by Mathematical Induction

To demonstrate $P \Rightarrow Q$ by induction we require that the truth of P and Q be expressed as a function of some ordered set S.

1. *(Basis)* Show that $P \Rightarrow Q$ is valid for a specific element k in S.
2. *(Inductive Hypothesis)* Assume that $P \Rightarrow Q$ for some element n in S.
3. Demonstrate that $P \Rightarrow Q$ for the element $n + 1$ in S.
4. Conclude that $P \Rightarrow Q$ for all elements greater than or equal to k in S.

https://www.cse.wustl.edu/~cytron/547Pages/f14/IntroToProofs_Final.pdf

Proof by Mathematical Induction

1. (Basis) Show that it is **valid for n = 1** in $\mathbb{Z}^+$.

2. (Inductive Hypothesis) Assume that it is true **for some element n** in $\mathbb{Z}^+$.

3. (Induction) Demonstrate that it is true **for the element n + 1** in $\mathbb{Z}^+$.

4. Conclude that it is true **for all elements greater than or equal to 1** in $\mathbb{Z}^+$.

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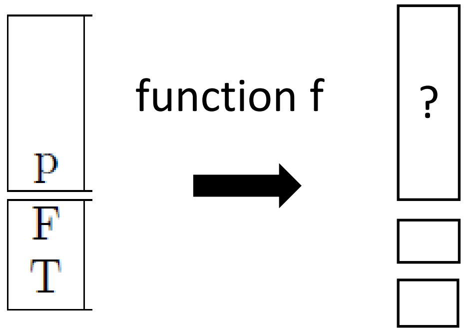
http://www.mathcentre.ac.uk/resources/uploaded/mathcentre-proof2.pdf

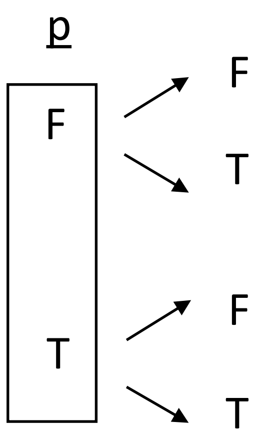
Proof by Mathematical Induction

1. (Basis) Show that it is **valid for n = 1** in $\mathbb{Z}^+$.

\# of functions for 1 variable $= 2^{2^1} = 4$

1 variable: p





p	$f_0 = F$	$f_1 = p$	$f_2 = \neg p$	$f_3 = T$
F	F	F	T	T
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TABLE 1. Boolean functions of one variable

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Proof by Mathematical Induction

2. (Inductive Hypothesis) Assume that it is true **for some element n** in $\mathbb{Z}^{+}$.

$$Assume\ that\ a_n = 2^{(2^n)}\ is\ true\ for\ some\ n\ in\ \mathbb{Z}^{+}$$

3. (Induction) Demonstrate that it is true **for the element n + 1** in $\mathbb{Z}^{+}$.

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Proof by Mathematical Induction

3. (Induction) Demonstrate that it is true **for the element n + 1** in $\mathbb{Z}^+$.

2 variables: p and q

p	q
F	F
F	T
T	F
T	T

$$\#\ of\ rows\ for\ n\ variables = 2^n$$

$$\#\ of\ rows\ for\ n+1\ variables = 2^{n+1}$$

In general, adding 1 variable: (n+1) case

r	p	q
F	F	F
F	F	T
F	T	F
F	T	T
T	F	F
T	F	T
T	T	F
T	T	T

functions

F
T
F
T

$$a_{n+1} = 2^{(2^{n+1})}$$

Proof by Mathematical Induction

4. Conclude that it is true **for all elements greater than or equal to 1** in $\mathbb{Z}^{+}$.

$$a_n = 2^{(2^n)} \; for \; n \geq 1 \; in \; \mathbb{Z}^{+}$$

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n	2^n	$2^{(2^n)}$

$r = 2^n$

2^r

Thank you for listening.